2.4

Simulation of Random Samples

Contents

1 Introduction

This project is programmed in **MATLAB R2014a**. Consult section A for program documentation, listings and information on the structure of the programming for the project. This report is written in $\LaTeX\,2_\varepsilon$.

2 The Exponential Distribution

Q1. Fixing $\theta \in (0, \infty)$, the rate of the distribution, the distribution function $F_\theta : [0, \infty) \to [0, 1)$ is a bijection. The median is thus $m(\theta) = F_\theta^{-1}(1/2)$:

$$F_\theta(m) = 1 - e^{-\theta x} = \frac{1}{2} \implies m(\theta) = \frac{\ln(2)}{\theta}$$

The function $m : (0, \infty) \to (0, \infty)$, $\theta \mapsto m(\theta)$ is thus also a bijection, so has inverse

$$\theta(m) = \frac{\ln(2)}{m}$$

expressing the rate of the distribution in terms of its median. Hence, $\forall m \in (0, \infty)$

$$f\left(x|\theta(m)\right) = \theta(m)e^{-\theta(m)x} = \frac{\ln(2)}{m}2^{-\frac{x}{m}}$$

Q2. The distribution being sampled from has median $m_0 = \ln(2)/1.2 = 0.5776$. The following is a sample $\{x_i\}_{i=1}^6$ of size 6 output by `A.E`:

$$0.1417 \quad 1.6122 \quad 0.8625 \quad 0.3934 \quad 0.1766 \quad 0.4659$$

The log likelihood of the median estimated from this sample, $\ell\left(m|\left\{x_i\right\}_{i=1}^6\right)$, is plotted below:

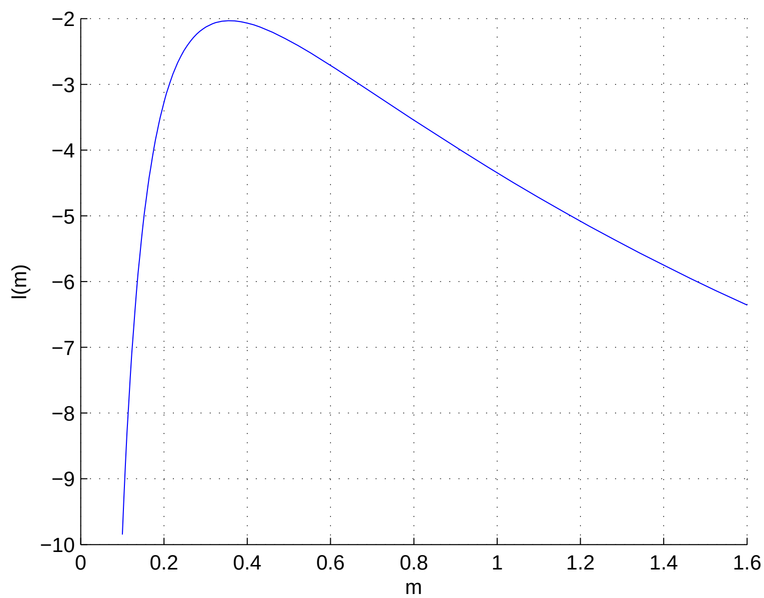


Figure 1: Likelihood of the median of a sample of size 6.

The maximum of the log likelihood provides an estimate for the median of the distribution. It can in fact be derived analytically, as follows. Given a sample $\{x_i\}_{i=1}^n$ from the exponential distribution with median m_0, the log likelihood function $\ell\left(\cdot|\left\{x_i\right\}_{i=1}^n\right) : (0, \infty) \to \mathbb{R}$ takes the form:

$$\ell(m) = \sum_{i=1}^n \ln\left(f\left(x_i|\theta(m)\right)\right) = \sum_{i=1}^n \ln\left(\frac{\ln(2)}{m}e^{\frac{-x_i \ln(2)}{m}}\right) = n\left(\ln^2(2) - \ln(m) - \frac{\ln(2)}{m}\overline{x}\right) \qquad (1)$$

It is smooth on $(0, \infty)$, with first and second derivatives:

$$\ell'(m) = n\left(-\frac{1}{m} + \frac{\ln(2)}{m^2}\overline{x}\right) \qquad \ell''(m) = \frac{n}{m^2}\left(1 - \frac{2\ln(2)}{m}\overline{x}\right)$$

Now, $\ell'(m) = 0 \iff m = \ln(2)\overline{x}$ and $\ell''(\ln(2)\overline{x}) = \frac{n}{\ln(2)^2\overline{x}^2}(1-2) = -\frac{n}{\ln(2)^2\overline{x}^2} < 0$,[1] so $\ln(2)\overline{x}$ is the unique preimage of the global maximum of the function. Thus, $\hat{m} = \ln(2)\overline{x}$. Indeed, treating this median estimator as a random variable $\hat{M}$ defined in terms of the sample mean $\overline{X}$, $\mathbb{E}(\hat{M}) = \ln(2)\mathbb{E}(\overline{X}) = \frac{\ln(2)}{\theta_0} = m_0$, so it is an unbiased estimator. The estimate yielded by the above example sample is $\hat{m} = 0.3578$, in this case less than the distribution median 0.5776.

Q3. Here are plots of median log likelihood functions for larger sample sizes n:

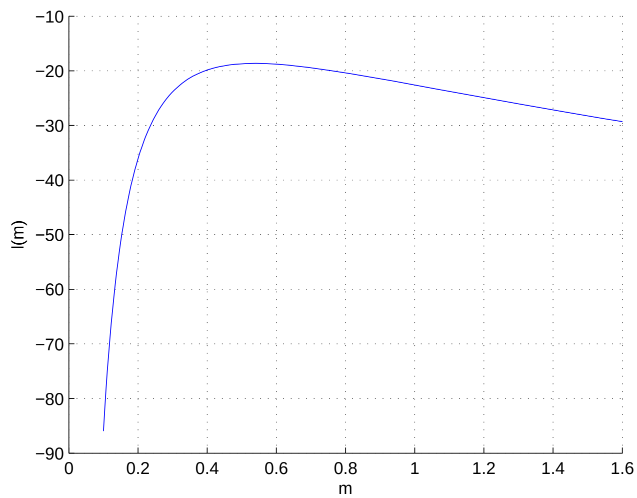


Figure 2: $n = 25$; $\hat{m} = 0.5375 < m_0$.

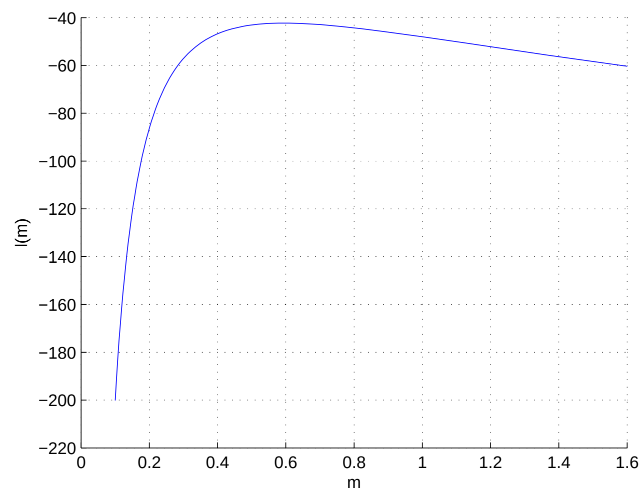


Figure 3: $n = 50$; $\hat{m} = 0.5939 > m_0$.

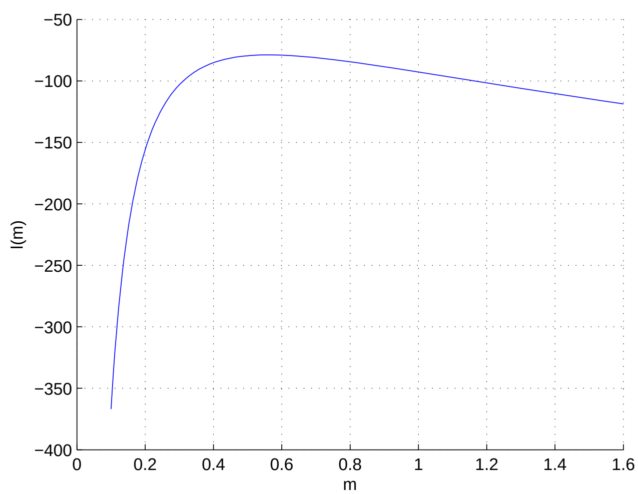


Figure 4: $n = 100$; $\hat{m} = 0.5605 < m_0$.

The graphs have broadly the same shape, the only significant difference being their extent along the y-axis, which increases with n. This is expected as the likelihood function, as in equation (1), is directly proportional to n and linear in $\overline{x}$ (fixing the other variable), and depends on nothing else. By the central limit theorem, $\sqrt{n}\frac{\overline{X}-\mu}{\sigma}$ converges in distribution as $n \to \infty$ to a standard normal distribution; in particular, it becomes increasingly likely that a sample will yield a sample mean (and so median estimate) that is at most a fixed distance away from the true parameter, and so it is less likely that there will be variation in the shape of the graphs.[2]

[1] Here and in Q6, $\mathbb{P}(\overline{X} = 0) = 0$, so the case $\overline{x} = 0$ is theoretically negligible and computationally unlikely.

[2] More precisely, considering $\frac{1}{n}\ell$ as a (scaled) function $\frac{1}{n}\ell(m, \overline{x})$, on a compact subset $A \subset (0, \infty)$, $\mathcal{L} : (0, \infty) \to C(A)$, $\overline{x} \mapsto \frac{1}{n}\ell(\cdot, \overline{x})$ is continuous w.r.t. absolute value on the domain and the uniform norm on the range. Thus, small changes in $\overline{x}$ produce uniformly small visible changes in the graph.

Q4. For $\lambda < \theta$ (a neighbourhood of 0),

$$M_X(\lambda) = \int_0^\infty e^{\lambda x}\theta e^{-\theta x}\mathrm{d}x = \theta \int_0^\infty e^{(\lambda-\theta)x}\mathrm{d}x = \frac{\theta}{\lambda - \theta}\left[e^{(\lambda-\theta)x}\right]_{x=0}^\infty = \frac{1}{1-\frac{\lambda}{\theta}}$$

X, Y are independent, so $M_{X+Y}(\lambda) = M_X(\lambda)M_Y(\lambda) = \frac{1}{\left(1-\frac{\lambda}{\theta}\right)^2}$ on the same neighbourhood of 0. This agrees with the MGF of a $\Gamma(2,\theta)$ random variable, so $X+Y \sim \Gamma(2,\theta)$, as the distribution of a random variable is determined by its MGF, if the latter exists.

Q5. The distribution function of a $\Gamma(2,\theta)$ random variable ($\theta \in (0,\infty)$) takes this form:

$$\begin{aligned} F(x) &= \theta^2 \int_0^x te^{-\theta t}\mathrm{d}t = \theta^2 \left(\left[t\left(-\frac{1}{\theta}e^{-\theta t}\right)\right]_{t=0}^x - \int_0^x \left(-\frac{1}{\theta}\right)e^{-\theta t}\mathrm{d}t\right) \\ &= \theta^2\left(-\frac{1}{\theta}xe^{-\theta x} + \frac{1}{\theta}\left[-\frac{1}{\theta}e^{-\theta t}\right]_{t=0}^x\right) = -\theta x e^{-\theta x} - \left(e^{-\theta x} - 1\right) = 1 - (1+\theta x)\,e^{-\theta x} \end{aligned}$$

$F : [0,\infty) \to [0,1)$ is a bijection with no obvious closed-form inverse. Therefore, it may be more computationally efficient to generate variables from this distribution by generating sums of two independent $\mathrm{Exp}(\theta)$ variables. This method is used by the function `A.G`.

Q6. Given a sample $\{x_i\}_{i=1}^n$ from a $\Gamma(2,\theta)$ distribution, the log likelihood function for the rate θ, $\ell\left(\cdot|\,\{x_i\}_{i=1}^n\right) : (0,\infty) \to \mathbb{R}$, takes the form:

$$\ell(\theta) = \sum_{i=1}^n \ln\left(\theta^2 x_i e^{-\theta x_i}\right) = \sum_{i=1}^n \left(2\ln(\theta) + \ln(x_i) - \theta x_i\right) = n(2\ln(\theta) + \overline{\ln(x)} - \theta\overline{x}) \tag{2}$$

It is smooth on $(0,\infty)$, with first and second derivatives:

$$\ell'(\theta) = n\left(\frac{2}{\theta} - \overline{x}\right) \qquad \ell''(\theta) = -\frac{2n}{\theta^2}$$

Now, $\ell'(\theta) = 0 \iff \theta = \frac{2}{\overline{x}}$ and $\ell''\left(\frac{2}{\overline{x}}\right) = -\frac{1}{2}n\overline{x}^2 < 0$, so $\frac{2}{\overline{x}}$ is the unique preimage of the global maximum of the function. Thus, $\hat{\theta} = \frac{2}{\overline{x}}$.

Let θ_0 be the true rate of the distribution. The sum of n $\Gamma(2,\theta_0)$ random variables X_i is distributed as $\Gamma(2n,\theta_0)$.[3] This fact allows the expectation of the estimator, as a random variable $\hat{\Theta}$, to be calculated:

$$\mathbb{E}(\hat{\Theta}) = 2n\mathbb{E}\left(\frac{1}{\sum X_i}\right) = 2n\int_0^\infty \frac{\theta_0^{2n}}{(2n-1)!}t^{2n-2}e^{-\theta_0 t}\mathrm{d}t = \left(1 + \frac{1}{2n-1}\right)\theta_0 \tag{3}$$

The latter integral is evaluated by comparison with the density of a $\Gamma(2n-1)$ random variable. This shows that the estimator is biased (tending to overestimate θ_0) but asymptotically unbiased (i.e. $\mathbb{E}(\hat{\Theta}) \to \theta_0$ as $n \to \infty$).

[3] By a similar argument to Q4.

Q7. Here are plots of log likelihood functions (of the rate of $\Gamma(2, \theta)$) for different sample sizes. The true rate is $\theta_0 = 2.2$.

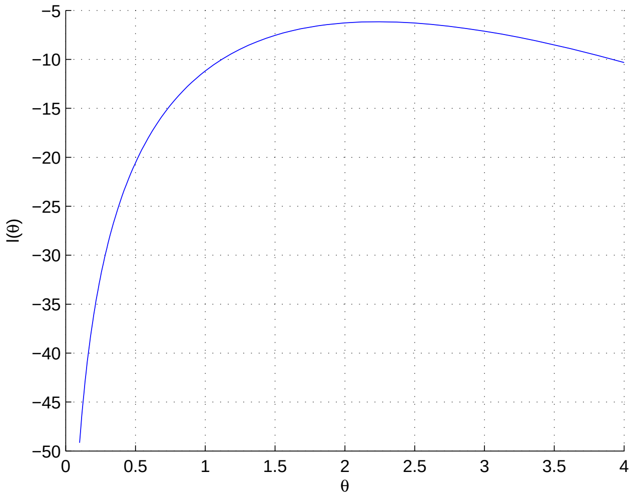


Figure 5: $n = 10$; $\hat{\theta} = 2.2326 > \theta_0$

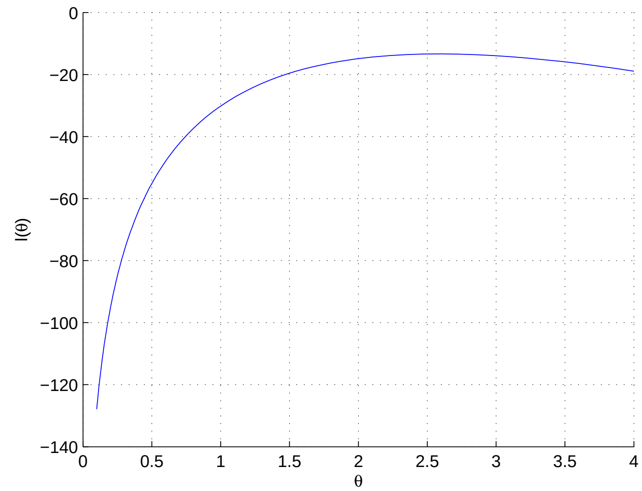


Figure 6: $n = 25$; $\hat{\theta} = 2.5833 > \theta_0$

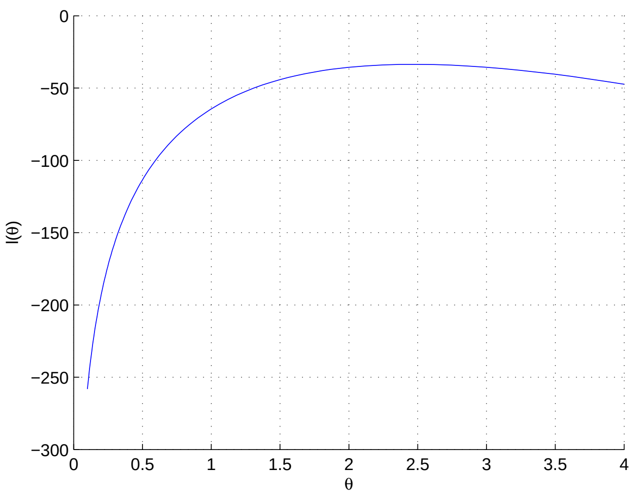
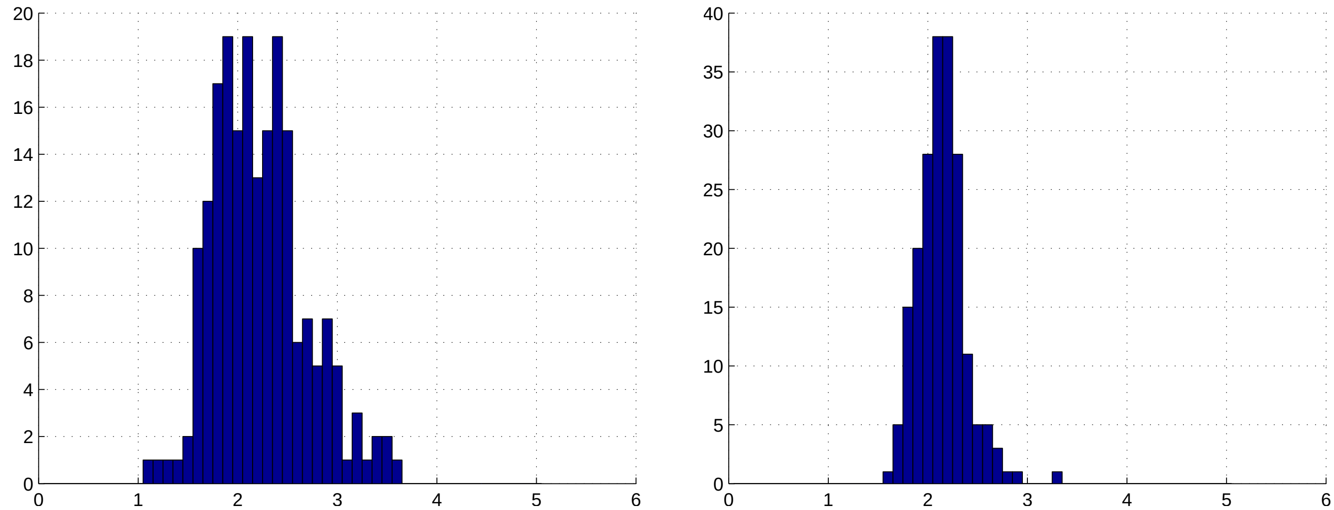


Figure 7: $n = 50$; $\hat{\theta} = 2.4676 > \theta_0$

This log likelihood function, given by equation (2), is again directly proportional to n and depends linearly on $\overline{x}$. It also depends, as an added constant, on $\overline{\ln(x)}$ (once rescaled by dividing by n), but this shift doesn't affect the shape of the graph. $\overline{x}$ converges by the central limit theorem and so the shape of the graph (once rescaled) converges as the sample size increases.

However, unlike with the exponential median, for any fixed sample size, it is not expected that the maximum likelihood estimator of the rate will give the true rate; indeed, the expected estimate, given by equation (3), is always greater than the true value, though the discrepancy tends to 0 as $n \to \infty$. In the case of the samples yielding these example graphs, the biased estimator has always overestimated the rate. Its expected value, for $n = 10, 25, 50$, is respectively 2.3158, 2.2449, 2.2222.

Q8. The following histograms represent samples of size 200 taken from the sampling distribution of the mean of samples of size n of a $\Gamma(2, 2.1)$ random variable. As $N \to \infty$, these will take on the shape of the probability density function of the random variable.



(a) $n = 10$. Mean: 2.2211.

(b) $n = 40$. Mean: 2.1402.

Figure 8: The sampling distributions of the mean of n samples of $\Gamma(2, 2.1)$.

By equation (3), the expected means of the $n = 10$ and $n = 40$ distributions are respectively 2.2105 and 2.1266. The variances of the distributions converge, by the central limit theorem, to 0 as $N \to \infty$. This corresponds with the observations made from the graphs (of individual samples from the sampling distributions) – the samples have means as stated in the captions and appear to be more concentrated around the mean for $n = 40$ than for $n = 10$, as $10 < 40$.[4]

[4] Indeed, since large samples are being taken to estimate the expected value of the estimator in this question, there is less variance in their means between instances of the script `Q8` than there is in the 1-sample estimates of Q7, for the same reason (applying the central limit theorem to the sampling distribution of $\hat{\Theta}_{200}$).

3 The Normal Distribution

Q9. The function $\alpha : [0, 2\pi) \times (0, \infty) \to \mathbb{R}^2 \setminus (\mu_1, \mu_2)$, $(\phi, v) \mapsto (x(\phi, v), y(\phi, v))$ is a bijection (it is a scaled polar parameterisation). Let $\alpha^{-1} : (x, y) \mapsto (\phi(x, y), v(x, y))$. Then $\forall (x, y) \in \mathbb{R}^2 \setminus (\mu_1, \mu_2)$

$$v(x,y) = v(x,y)\cos(\phi(x,y))^2 + v(x,y)\sin(\phi(x,y))^2 = \frac{1}{\sigma^2}\left((x-\mu_1)^2 + (y-\mu_2)^2\right)$$

The partial derivatives of α (identifying 0 with 2π in the ϕ component) are as follows:

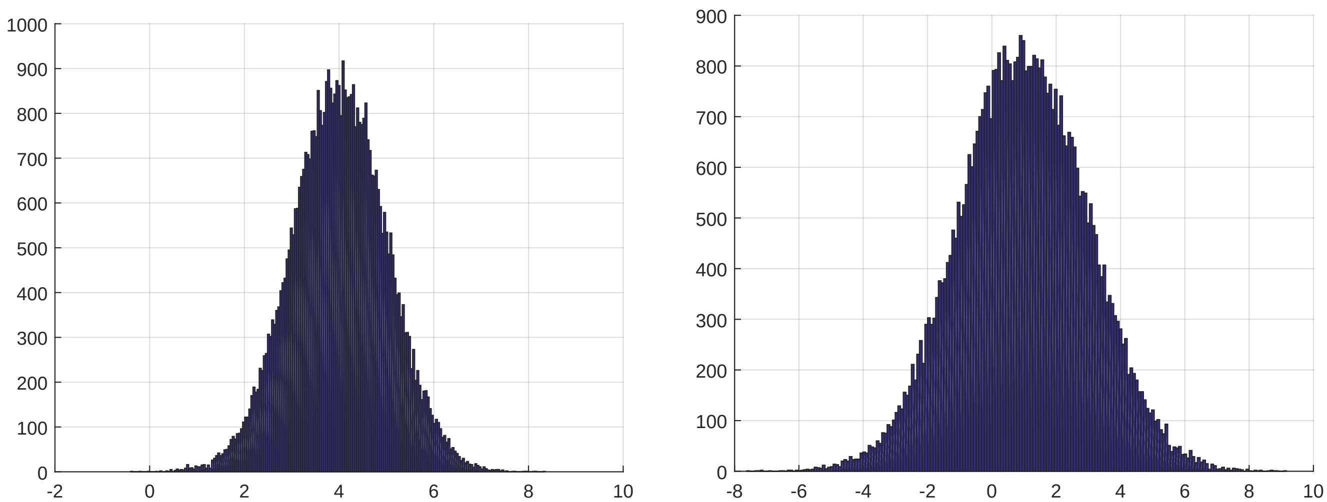
$$\frac{\partial(x,y)}{\partial(\phi,v)} = \begin{vmatrix} -\sigma\sqrt{v}\sin(\phi) & \frac{\sigma}{2\sqrt{v}}\cos(\phi) \\ \sigma\sqrt{v}\cos(\phi) & \frac{\sigma}{2\sqrt{v}}\sin(\phi) \end{vmatrix} = \frac{\sigma^2\sqrt{v}}{2\sqrt{v}}\begin{vmatrix} -\sin(\phi) & \cos(\phi) \\ \cos(\phi) & \sin(\phi) \end{vmatrix} = -\frac{\sigma^2}{2}$$

whence (X, Y) have joint density $g : \mathbb{R}^2 \to [0, \infty)$, where $\forall (x, y) \in \mathbb{R}^2 \setminus (\mu_1, \mu_2)$,[5]

$$g(x,y) = f\left(\phi(x,y), v(x,y)\right)\left|\frac{\partial(x,y)}{\partial(\phi,v)}\right|^{-1} = \frac{1}{2\pi\sigma^2}e^{-\frac{1}{2}v(x,y)} = \left(\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_1)^2}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(y-\mu_2)^2}\right)$$

Thus, $X \sim N(\mu_1, \sigma^2)$, $Y \sim N(\mu_2, \sigma^2)$ and X, Y are independent (as the density function factorises).

Q10. Normally-distributed random samples are produced by `A.N`.



(a) $\mu = 4$, $\sigma = 1$ (b) $\mu = 1$, $\sigma = 2$

Figure 9: Normally-distributed random samples with sample size 50,000.

Fixing a sample size n, the sample mean $\overline{X}$ of n independent $N(\mu, 1)$ variables has the distribution $N(\mu, \frac{1}{n})$. Furthermore, $\sqrt{n}\left(\overline{X} - \mu\right) \sim N(0, 1)$. Let Φ be the distribution function of $N(0, 1)$ so that $\mathbb{P}\left(\sqrt{n}\left|\overline{X} - \mu\right| \leq \Phi^{-1}(0.9)\right) = 0.8$. Then

$$0.8 = \mathbb{P}\left(-\Phi^{-1}(0.9) \leq \sqrt{n}\left(\mu - \overline{X}\right) \leq \Phi^{-1}(0.9)\right) = \mathbb{P}\left(\overline{X} - \frac{\Phi^{-1}(0.9)}{\sqrt{n}} \leq \mu \leq \overline{X} + \frac{\Phi^{-1}(0.9)}{\sqrt{n}}\right)$$

Thus, the 80% confidence interval given by $\overline{X}$ is the symmetric interval

$$I_{\overline{X}} = \left[\overline{X} - \frac{\Phi^{-1}(0.9)}{\sqrt{n}}, \overline{X} + \frac{\Phi^{-1}(0.9)}{\sqrt{n}}\right]$$

[5] The value of $g(\mu_1, \mu_2)$ is inconsequential, as it doesn't affect any integrals of g.

Treating it as an interval-valued random variable derived from $\overline{X}$, it has the property that $\mathbb{P}\left(\mu \in I_{\overline{X}}\right) = 0.8$.

Q11. Taking samples 25 times, it is expected that the 80% confidence interval for the sample mean $I_{\overline{x}}$ will contain the true mean μ $0.8 \times 25 = 20$ times (and so not contain it 5 times). An experiment follows, with samples of size $n = 100$ from the $N(0, 1)$ distribution. The confidence intervals are denoted $[\overline{x} - c, \overline{x} + c]$, where $c = \frac{\Phi^{-1}(0.9)}{\sqrt{n}}$.

$\overline{x}$	$\overline{x} - c$	$\overline{x} + c$	$\mathbb{1}\left(\mu \in I_{\overline{x}}\right)$	$\overline{x}$	$\overline{x} - c$	$\overline{x} + c$	$\mathbb{1}\left(\mu \in I_{\overline{x}}\right)$
0.1743	0.0461	0.3024	0	-0.2001	-0.3283	-0.0719	0
-0.0600	-0.1881	0.0682	1	0.1268	-0.0013	0.2550	1
0.1448	0.0167	0.2730	0	0.1031	-0.0250	0.2313	1
0.0477	-0.0804	0.1759	1	-0.1147	-0.2428	0.0135	1
0.1431	0.0149	0.2712	0	0.1196	-0.0086	0.2478	1
-0.1023	-0.2305	0.0258	1	-0.2090	-0.3372	-0.0808	0
0.1565	0.0284	0.2847	0	0.1699	0.0418	0.2981	0
-0.0327	-0.1609	0.0954	1	-0.0897	-0.2179	0.0385	1
0.0018	-0.1264	0.1300	1	-0.0697	-0.1978	0.0585	1
-0.0642	-0.1923	0.0640	1	0.0656	-0.0626	0.1937	1
0.0302	-0.0979	0.1584	1	0.0082	-0.1200	0.1363	1
-0.0526	-0.1807	0.0756	1	0.0442	-0.0839	0.1724	1
0.0715	-0.0566	0.1997	1				

Figure 10: The confidence interval did not contain μ 7 times, in this case.

Q12. For each sample $\{X_i\}_{i=1}^{n}$, $\mathbb{1}\left(\mu \notin I_{\overline{X}}\right) \sim B(1, 0.2)$. Thus, independently sampling 25 times, as each indicator $\mathbb{1}\left(\mu \in I_{\overline{X}}\right)$ is independent of the others (independence being preserved by functions of random variables), the number of times the confidence interval does not contain μ, denoted Y, has distribution $B(25, 0.2)$. This depends on the number of samples taken and the confidence level, but not on the individual sample size nor indeed on the distribution being sampled from. Therefore, sampling with $n = 50$ and $\mu = 4$, we have $\mathbb{E}(Y) = 5$ and indeed $\mathrm{Var}(Y) = 4$, unchanged from the case in the previous question. However, there is an advantage to increasing n: the size of the confidence interval, $\frac{2\Phi^{-1}(0.9)}{\sqrt{n}}$, is reduced, so μ can be estimated more precisely. Equivalently, the same-sized confidence interval gives estimates to a higher level of confidence (than, e.g., 80%).

4 The χ^2 Distribution

Q13. The χ_d^2 distribution is plotted below with samples of size 100, 250, 500 respectively from left to right.

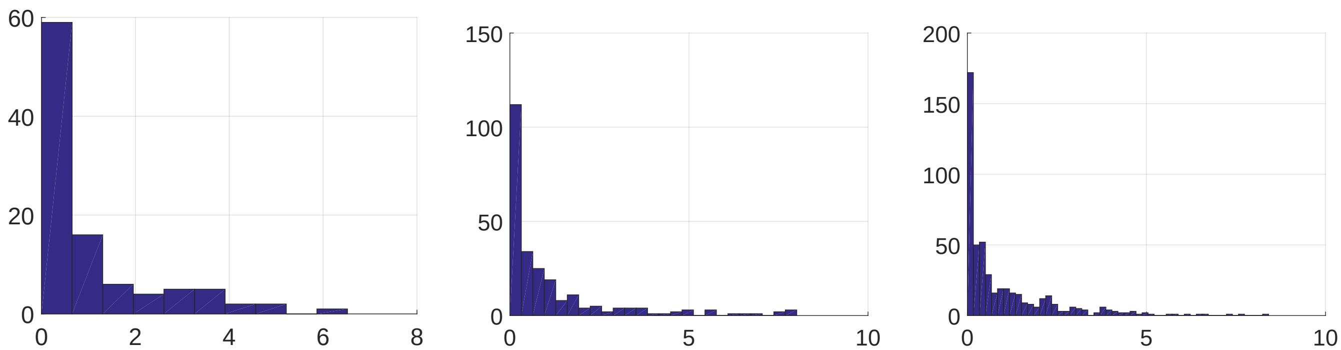


Figure 11: $d = 1$

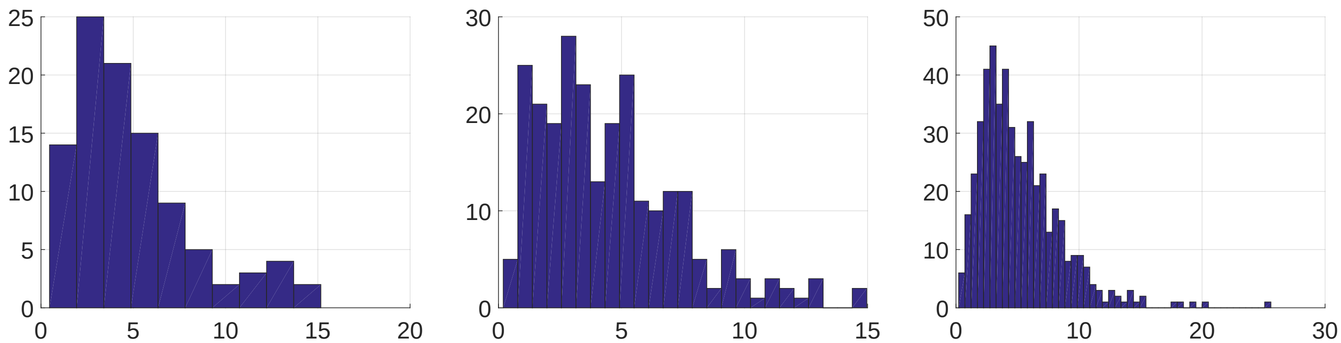


Figure 12: $d = 5$

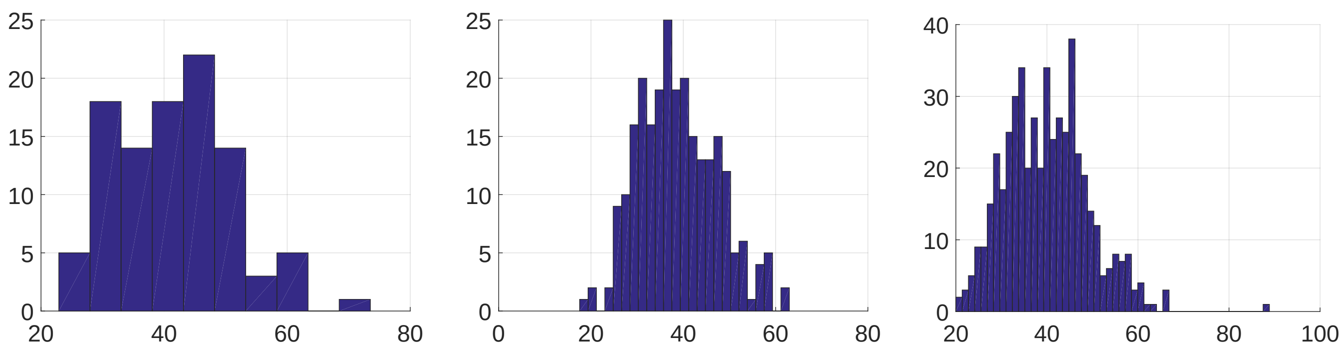


Figure 13: $d = 40$

As the degrees of freedom of the distribution increase, its mean and median increase (represented by a rightward shift of the apparent centre of the histograms) and its variance increases (as each histogram is programmed to have $\frac{n}{10}$ bins, which increase in width along the x-axis as d increases). Furthermore, it becomes increasingly symmetric.

This agrees with the theoretical behaviour of the distribution, which, for degree d, has mean d, variance $2d$ and skewness $\sqrt{8/d}$.[6]

[6]See, for example, http://mathworld.wolfram.com/Chi-SquaredDistribution.html (results (37), (39), (40); access date: 26/04/2015). Moments can be deduced from the coefficients of the moment generating function of the distribution.

A Programs

The programs take the form of a module, `A.m`, which provides methods for the project, and six scripts, `Q*.m`, which use these methods to generate output for each question. These depend on two auxiliary, unlisted scripts that process output and are based on scripts from the internet:

`tabler(L,d,f)`: Appends the table at global variable T in LaTeX-format to the file L.txt with heading d and formatting f, specified as a cell array of statements in MATLAB fprintf format.

`grapher(P)`: Writes the figure at global variable G in pdf-format to the file P.pdf.

A.1 Documentation

This section describes the purpose of the project's functions, and the inputs for which they are designed to give valid output.

`A.E(t,m,n,o)`: $\mathtt{t} \in (0, \infty)$, $\mathtt{m}, \mathtt{n}, \mathtt{o} \in \mathbb{N}$. Generates an $\mathtt{m} \times \mathtt{n} \times \mathtt{o}$ array of $\mathrm{Exp}(\mathtt{t})$ random variables.

`A.G(t,m,n)`: $\mathtt{t} \in (0, \infty)$, $\mathtt{m}, \mathtt{n} \in \mathbb{N}$. Generates an $\mathtt{m} \times \mathtt{n}$ matrix of $\Gamma(2, \mathtt{t})$ random variables.

`A.N(mu,s2,m,n)`: $\mathtt{mu} \in \mathbb{R}$, $\mathtt{s2} \in (0, \infty)$, $\mathtt{m}, \mathtt{n} \in \mathbb{N}$. Generates an $\mathtt{m} \times \mathtt{n}$ matrix of $N(\mathtt{mu}, \mathtt{s2})$ random variables.

`A.X(d,n)`: $\mathtt{d}$, $\mathtt{n} \in \mathbb{N}$. Generates an $\mathtt{n}$-vector of $\chi^2_{\mathtt{d}}$ random variables.

`A.ELM(m,d)`: $\mathtt{m} \in (0, \infty)$, $\mathtt{d} \in \bigcup_{n=1}^{\infty} [0, \infty)^n$. Evaluates $\ell(\mathtt{m}|\mathtt{d})$, the median log likelihood function of the exponential distribution given a sample $\mathtt{d}$.

`A.GLT(t,d)`: $\mathtt{t} \in (0, \infty)$, $\mathtt{d} \in \bigcup_{n=1}^{\infty} [0, \infty)^n$. Evaluates $\ell(\mathtt{t}|\mathtt{d})$, the rate log likelihood function of the Gamma distribution of shape 2 given a sample $\mathtt{d}$.

A.2 A

```
classdef A
    methods(Static)
        function x = E(t,m,n,o)
            x = -log(1-rand(m,n,o))/t;
        end
        function x = G(t,m,n)
            x = sum(A.E(t,m,n,2),3);
        end
        function x = N(mu,s2,m,n)
            f = 2*pi*rand(m,n);     v = -2*log(1-rand(m,n));
            x = mu + sqrt(s2)*sqrt(v).*cos(f);
        end
        function x = X(d,n)
            x = A.N(0,1,d,n); x = sum(x.*x,1);
        end
```

```
        function y = ELM(m,d)
            t = log(2)/m;
            g = @(x) log(t*exp(-x*t));
            y = sum(arrayfun(g,d));
        end
        function y = GLT(t,d)
            g = @(x) log((t^2)*x*exp(-x*t));
            y = sum(arrayfun(g,d));
        end
    end end
```

A.3 Q2

```
T = 1.2; M = log(2)/T;
disp(strcat('m_0=',num2str(M)))                          % True median
for n = [6, 25, 50, 100]
    disp(strcat(num2str(n),':'))
    x = A.E(T,1,n,1)                                     % Sample
    e = log(2)*mean(x)                                   % Estimator
    l = @(m) A.ELM(m,x);
    G = figure; clf; hold all; grid on; axis on         % Graph
        fplot(l,[0.1,2])
        xlabel('m'); ylabel('l(m)'); xlim([0,1.6]);
    grapher(strcat('P2-',num2str(n)))
end
```

A.4 Q7

```
T = 2.2;
disp(strcat('rate=',num2str(T)))                         % True rate
for n = [10,25,50]
    x = A.G(T,1,n);
    disp(strcat(num2str(n),':',num2str(2/mean(x)))) % Estimator
    l = @(t) A.GLT(t,x);
    G = figure; clf; hold all; grid on; axis on         % Graph
        fplot(l,[0.1,4])
        xlabel('\theta'); ylabel('l(\theta)'); xlim([0,4]);
    grapher(strcat('P7-',num2str(n)))
end
```

A.5 Q8

```
t = 2.1; N = 200;
for n = [10,40]
    x = A.G(t,n,N);
    m = 2./mean(x);
    e = mean(m)                                          % Mean estimator
    G = figure; clf; hold all; grid on; axis on % Graph
        hist(m,0.1:0.1:5)
    grapher(strcat('P8-',num2str(n)))
end
```

A.6 Q10

```
for a = [4,1;1,4]
    b = num2cell([a',1,50000]);
    x = A.N(b{:});
    G = figure; clf; hold all; grid on; axis on % Graph
        hist(x,200)
    grapher(strcat('P10-',num2str(a(1)),'-',num2str(a(2))))
end
```

A.7 Q11

```
mu = 0; n = 100; m = 25;
c = norminv(0.9,0,1)/sqrt(n);
x = A.N(0,1,n,m);
sm = mean(x)';  lb = sm - c;
ub = sm + c;    ci = abs(sm-mu) < c;
T = table(sm,lb,ub,ci)                          % Table
d = sum(ci == 0)                                % Times not contained
tabler('L11','',{'%5.4f',3,'%1.0f',1})
```

A.8 Q13

```
for i = [1,5,40]
for j = [100,250,500]
    x = A.X(i,j);
    G = figure; clf; hold all; grid on; axis on % Graph
        hist(x,round(j/10)); set(gca,'fontsize',20)
    grapher(strcat('P13-',num2str(i),'-',num2str(j)))
end
end
```